It is this formula which is usually referred to as *Cauchy's integral formula.* We must remember that it is valid only when $n(\gamma, z) = 1$, and that we have proved it only when $f(z)$ is analytic in a disk.

EXERCISES

1. Compute

$$\int_{|z|=1} \frac{e^z}{z}\, dz.$$

2. Compute

$$\int_{|z|=2} \frac{dz}{z^2+1}$$

by decomposition of the integrand in partial fractions.

3. Compute

$$\int_{|z|=\rho} \frac{|dz|}{|z-a|^2}$$

under the condition $|a| \neq \rho$. *Hint:* make use of the equations $z\bar{z} = \rho^2$ and

$$|dz| = -i\rho \frac{dz}{z}.$$

2.3. Higher Derivatives. The representation formula (22) gives us an ideal tool for the study of the local properties of analytic functions. In particular we can now show that an analytic function has derivatives of all orders, which are then also analytic.

We consider a function $f(z)$ which is analytic in an arbitrary region Ω. To a point $a \in \Omega$ we determine a δ-neighborhood Δ contained in Ω, and in Δ a circle C about a. Theorem 6 can be applied to $f(z)$ in Δ. Since $n(C,a) = 1$ we have $n(C,z) = 1$ for all points z inside of C. For such z we obtain by (22)

$$f(z) = \frac{1}{2\pi i} \int_C \frac{f(\zeta)\, d\zeta}{\zeta - z}.$$

Provided that the integral can be differentiated under the sign of integration we find

$$f'(z) = \frac{1}{2\pi i} \int_C \frac{f(\zeta)\, d\zeta}{(\zeta - z)^2} \tag{23}$$

and

$$f^{(n)}(z) = \frac{n!}{2\pi i} \int_C \frac{f(\zeta)\, d\zeta}{(\zeta - z)^{n+1}}. \tag{24}$$